# AIDT
## PRE-ALGEBRA MANUAL
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I. PRE-ALGEBRA

A. WHAT IS ALGEBRA?

Learner Objectives: Upon completion of this unit, the student will have a general concept of how Algebra takes math to the next level. Students will learn the terminology and symbols of basic Algebra while incorporating the concepts used in Basic Math.

Algebra is a branch of mathematics in which arithmetic relations are generalized and explored by using letter symbols to represent numbers, variable quantities, and other mathematical entities.


Today, algebra is the study of the properties of operations on numbers. Algebra generalizes arithmetic by using symbols, usually letters, to represent numbers or unknown quantities. Algebra is a problem-solving tool; like a tractor or plow is a farmer's tool, algebra is the mathematician's tool for solving problems. Algebra has applications to every human endeavor. From art to medicine to zoology, algebra can be a tool. People who say that they will never use algebra are people who do not know algebra. Learning algebra is a bit like learning to read and write. If you truly learn algebra, you will use it. Knowledge of algebra can give you more power to solve problems and accomplish what you want in life.

By now, you might think that algebra must be complicated. Yes, it takes work, but algebra is something everyone can learn. In fact, you have probably already done some algebra in elementary school! Do you remember seeing problems like 5 + ? = 8? Well, this is really an algebraic equation. The only difference is that we will write a letter such as x instead of a "?" for the unknown number. So we write the
above equation as $5 + x = 8$. Next, we could try to solve the equation. In other words, we could try to find a number which makes the equation work. $5 + \text{what} = 8$? Yes, $x = 3$ works since $5 + 3 = 8$. So we say that 3 is the solution to the equation $5 + x = 8$. Algebra really just grows from this basic idea. Algebra helps you write and solve equations.

Since we use letters in algebra, we will rarely use $\times$ (St. Andrew's cross) to indicate multiplication. Around 1692, Leibniz used a raised dot, $\cdot$, instead of an "$\times$" to avoid confusion with the letter "x." Sometimes we'll write a dot for multiplication, but usually we'll just write nothing between letters to indicate multiplication! This can be confusing at first, so take note of the following ways of writing the expression "a number $r$ times another number $s$.”

$r$ times $s$ may be written in any of the following ways:

$$r \cdot s \quad rs \quad (r)(s) \quad r(s) \quad (r)s$$

This module of study is entitled "Pre-Algebra" because it introduces only the most basic concepts and rules, and sets the groundwork for more advanced study, should you have the need or desire to pursue it.

1. **Integers**

In Basic Math, you learned to solve problems using the Arabic number system, with the numerals (0, 1, 2, 3, 4, 5, 6, 7, 8, 9). These 10 digits made up all of the whole numbers and fractions that you encountered.

Algebra explores some different uses of these digits and so it is necessary to give them a different name, INTEGERS. Integers are the set of all whole numbers, both positive and negative and including zero. The concept of integers is more easily understood if you think about the number line introduced in Basic Arithmetic.

The number line was used to demonstrate the position and value of whole numbers, but only dealt with the positive side of the line.
FIGURE 1-1
Number Line

To understand integers, we have to extend the line in both directions, (positive and negative), with 0 at the starting point, as far as needed to include all the integers being considered. Each place on the number line represents an integer. So, the "integer line" will look like this:

FIGURE 1-2
Integer Line

The dots and the "infinity sign" at each end of the line means that the line can continue in either direction as far as required. So, every place on the line represents an integer, meaning that 0, 1, 27, -5, -936, 53, -2748 are all integers.

By expanding our concept of the number line, you can see how algebra is used to evaluate more complicated aspects of mathematics.

The number line for integer values includes:

- All the positive whole numbers
- Zero
- All the negative whole numbers

Negative-value integers are located to the left of the zero on the integer number line. Negative-value integers use the same symbols as the whole number system, but are distinguished by the use of a negative sign ( – ). Numbers 5 and – 5, for example, might resemble one
another in most respects, but they are two entirely different values. Refer to the integer number line, and you will see that 5 and –5 are located in two entirely different places.

Positive-value integers are located to the right of the zero on the integer number line. Positive integers are sometimes indicated with a positive sign (+). More often, however, we omit the positive sign. So when you see an integer value that does not have a sign, you can rightly assume it is a positive value.

A plus sign (+) is used for two entirely different purposes:
• to indicate the addition operation
• to indicate a positive integer value.

Likewise, a minus sign (–) is used for two entirely different purposes:
• to indicate the subtraction operation
• to indicate a negative integer value.

This can be confusing at times, but we all have to learn to live with the double meanings of the + and – signs.

Zero (0) has no sign.

2. **Comparing the Value of Integers**

The values of integers increase from left-to-right along the number line.
The values of integers decrease from right-to-left along the number line.
Examples:

4 > 1  
4 is greater than one. Why? Because 4 is located to the right of 1 on the integer number line.

0 < 3  
0 is less than 3. Why? Because 0 is located to the left of 3 on the integer number line.

– 1 < 4  
– 1 (minus-one) is less than 4. Why? Because –1 is located to the left of 4 (or + 4) on the integer number line.

– 3 > – 5  
– 3 (minus-three) is greater than minus – 5. Why? Because –3 is located to the right of –5 on the integer number line.

Notice in the examples above that the (<) or (>) sign is used to express "less than" or "greater than" values. In algebra, these expressions of value are frequently used. A helpful way to remember these values is to remember that the arrow always points to the smaller, or "less than" value. For example.

4 < 5  means "4 is less than 5".

3 > 2  means "3 is greater than 2".

– 8 < – 4  means "-8 is less than -4". (Remember, when working with "negative" integers, a number's value is determined by its place on the number line, not by the value of the whole number digit).

3. Absolute Values

Definition

The absolute value of an integer is its value without regard to the sign. Or to put it another way, the absolute value of an integer is its distance from the origin (zero) on the number line.
The absolute value of numbers is indicated by enclosing the numbers in a pair of vertical lines, | |.

For example, the absolute value of – 10 is written as | – 10 |.

**Examples:**

**What is the absolute value of -5?**
The distance between 0 and – 5 on the number line is 5 units.
Therefore the absolute value of – 5 is equal to 5.
\[ | -5 | = 5 \]

**What is the absolute value of 3?**
The distance between 0 and 3 on the number line is 3 units.
Therefore the absolute value of 3 is equal to 3.
\[ | 3 | = 3 \]

**What is the absolute value of zero?**
There is no distance between 0 and 0 on the number line, therefore the absolute value of 0 is 0.
\[ | 0 | = 0 \]
Understanding Integers Exercises

1. 27 is ___ 22
   - less than ( < )
   - greater than ( > )
   - equal to ( = )

2. 10 is ___ –6
   - less than ( < )
   - greater than ( > )
   - equal to ( = )

3. –37 is ___ 40
   - less than ( < )
   - greater than ( > )
   - equal to ( = )

4. Determine the absolute values.
   a. | – 20 | = __________
   b. | 25 | = __________
   c. | -1 | = __________
   d. | -129 | = __________
   e. | 0 | = __________
4. Adding Signed Integers

How to Add Signed Integers

The exact procedure for adding signed integers depends upon whether the addends have the same sign or opposite signs. "Addends is the name given to the numbers being added"

When the addends have the same sign (both + or both –):

Step 1: Add the absolute values of the addends.

Step 2. Give the result the sign that is common to the addends.

When the addends have opposite signs (one is + and the other is –):

Step 1: Subtract the absolute values of the addends

Step 2. Give the result the sign of the addend that has larger absolute value.

Adding Integers That Have the Same Sign

Adding integers that have the same sign means that both integers are positive or both are negative. For example:

Adding two positive integers: \((+2) + (+4) = \) ______

Adding two negative integers: \((-2) + (-4) = \) ______

To add integers that have the same sign (both positive or both negative):

Step 1: Add the absolute values of the addends

Step 2. Give the result the sign that is common to the addends
Adding Positive Integers

Example:

(+12) + (+7) =

Procedure:

• Add the absolute values of the addends.
  +12 | + | +7 | = 12 + 7 = 19

• Give the result the sign that is common to the addends. Both addends are positive, so the result is positive, +19

Solution:

(+12) + (+7) = ( +19 ) or 12 + 7 = 19

Note: There is no significant difference between adding a pair of positive integers and adding a pair of whole numbers.

A problem such as ( +6 ) + ( +4 ) = ( +10 ) can always be simplified to look like this: 6 + 4 = 10. And that looks exactly like a whole-number addition problem.

Adding Negative Integers

The rule for adding negative integers is the same as the rule for adding positive integers:

• Add the absolute values of the addends.

• Give the result the sign that is common to the addends.

In this case, the common sign is a negative sign.
Example:

\( (-12) + (-7) = ____ \)

- Add the absolute values of the addends.
  \[ | -12 | + | -7 | = 12 + 7 = 19 \]

- Give the result the sign that is common to the addends. Both addends are negative, so the result is negative, \(-19\)

Solution:  \(( -12 ) + ( -7 ) = ( -19 )\)  \text{ or }  \(-12) + ( -7 ) = -19\)

**Notes:** This is an addition problem. Although the addends both have negative values, you still add their absolute values.

Adding negative integers will always produce a negative sum.

A problem stated as \(( -6 ) + ( -18 ) = ( -24 )\) can be simplified a little bit: \(-6 + (-18 ) = -24\). It is not a good idea to remove the parentheses around the \(-18\), because the result would be confusing:
\[-6 + \ -18 = -24.\]

**Adding Integers That Have Opposite Signs**

Adding integers that have opposite signs means that one is positive and the other is negative. For example:

Adding a positive integer to a negative integer \((+2) + (-4) = \)
Adding a negative integer to a positive integer \((-2) + (+4) = \)

- Step 1: Subtract the absolute values.
- Step 2. Write the sum with the sign of the larger number.
Example 1:

\[( +6 ) + ( −4 ) = \underline{\phantom{0}}\]

• Subtract the absolute values
  \[| +6 | - | −4 | = 6 - 4\]

Write the sum with the sign of the larger number.
In this case, \(| +6 |\) is larger than \(| −4 |\), so the result is positive + 2

Solution: \(( +6 ) + ( −4 ) = ( +2 )\) or \(6 - 4 = 2\)

Example 2:

\[( −5 ) + ( +3 ) = \underline{\phantom{0}}\]

• Subtract the absolute values
  \[| −5 | - | +3 | = 5 - 3\]

• Write the sum with the sign of the larger number.
  \(| −5 |\) is larger than \(| +3 |\), so the result is negative - 2.

Solution: \(( −5 ) + ( +3 ) = ( −2 )\) or \(-5 + 3 = -2\)

**Integer Lesson Summary**

• To add integers that have the same sign (both positive or both negative):

  Step 1: Add the absolute values of the addends
  Step 2. Give the result the sign that is common to the addends

• To add integers that have opposite signs:

  Step 1: Subtract the absolute values.
  Step 2. Write the sum with the sign of the larger number.
5. **Subtracting Signed Integers**

There are two parts in the procedure for subtracting signed integers:

- Change the subtraction sign to the addition sign, and then switch the sign of the subtrahend (the number that immediately follows the operation sign you just changed).

- Add the result according to the procedures for adding signed integers.

**Changing Integer Subtraction to Integer Addition**

The purpose of the first step in this subtraction procedure is to change the operation from subtraction to addition. So you begin by changing the symbol of operation from subtraction to addition (from – to +), then switching the sign of the subtrahend (the number immediately following the operation sign you just changed).

![Image of a woman holding a megaphone with text: STOP !!!]

You have just been introduced to one of the most important and most critical rules you will ever encounter in the evaluation of algebraic equations. That is, **the changing of the symbol of operation and then switching the sign.**

As you study further into the world of algebra you will notice that practically all of the equations are formed around an equal (=) sign.

What that little sign is saying is that "Everything on the left side of this sign has the same value as everything on the right side". Both sides are "equal in value". They may not look alike, but they have the same value.

This is very important because the process of solving many algebraic equations requires moving some of the integers and signs from one
side of the "=" sign to the other and then changing the symbols of operation. Many of the rules of algebra are based on that principle. More on that later.

**Example 1:**

Here are some examples of changing integer subtraction to integer addition — only the first two steps in the procedure for subtracting signed integers.

1. \((+ 2) - (+ 6) = (+ 2) + (– 6)\)  
   Notice the change of symbol of operation

2. \((- 8) - (+ 11) = (- 8) + (– 11)\)  
   and integer sign.

3. \((+ 15) - (– 12) = (+ 15) + (+ 12)\)

4. \((- 2) - (– 7) = (– 2) + (+ 7)\)

Once you have converted the equations to addition problems, just follow the procedure for adding integers

**Completing the Operation**

**Subtracting Signed Integers**

Change the subtraction sign to the addition sign, and then switch the sign of the subtrahend (the number that immediately follows the operation sign you just changed.

Add the result according to the procedures for adding signed integers. It really is as simple as 1-2-3:
Example 2:

\((+15) - (+12) = ?\)

- Change from subtraction to addition:
- Change the subtraction sign to the addition sign.
- Then switch the sign of the number that immediately follows.

\((+15) - (+12) = (+15) + (-12)\)
\((+15) + (+12) = (+15) + (-12)\)

- Add the result

\((+15) + (-12) = (+3)\)

Solution:

\((+15) + (-12) = (+3)\) or \((15) + (-12) = 3\)

Example 3:

\((-8) - (-6) = ?\)

Procedure:

- Change from subtraction to addition:
- Change the subtraction sign to the addition sign.
- Then switch the sign of the number that immediately follows.

\((-8) - (-6) = (-8) + (-6)\)
\((-8) + (-6) = (-8) + (+6)\)

- Add the result

\((-8) + (+6) = (-2)\)

Solution:

\((-8) - (-6) = (-2)\)
6. Combining Integer Addition and Subtraction

Most of the examples and exercises in these lessons deal with only two terms at a time. However, you don’t have to advance much further in your study of mathematics until you encounter situations where you must add or subtract strings of three or more signed integers. For instance:

\[(+ 4) + (+ 5) + (+ 12) + (+ 6) = ?\]

\[(+ 12) + (– 14) + (– 8) = ?\]

\[(+ 2) – (+ 15) + (– 22) = ?\]

\[(+ 17) – (+ 24) + (– 1) – (+ 6) = ?\]

**Performing Combinations of Addition and Subtraction**

**Important:**

When adding or subtracting groups of three or more integers, always perform the operations from left to right.

**Example 1:**

\[(+ 4) + (+ 5) + (+ 12) + (+ 6) = ?\]

• Add the terms, two at a time, from left to right

\[(+ 4) + (+ 5) + (+ 12) + (+ 6) = (+ 9) + (+ 12) + (+ 6)\]

\[(+ 9) + (+ 12) + (+ 6) = (+ 21) + (+ 6)\]

\[(+ 21) + (+ 6) = (+ 27)\]

The Solution: \((+ 4) + (+ 5) + (+ 12) + (+ 6) = (+ 27)\)

The preceding example is just like a regular addition problem, where 
\(4 + 5 + 12 + 6 = 27\)

The parentheses and signs are being used to get you used to the practice of using them. They will be necessary as you proceed.
Example 2:

\[ (+12) + (-14) + (-8) = ? \]

Add the terms two at a time, from left to right.

\[ (+12) + (-14) + (-8) = (-2) + (-8) \]
\[ (-2) + (-8) = (-10) \]

The Solution: \[ (+12) + (-14) + (-8) = (-10) \]

When a series of three or more terms include subtraction operations as well as addition:

- Change the subtraction signs (–) to addition (+).
- Switch the sign attached to the term that follows the operation you just changed. If it's positive, change to negative. If it's negative, change to positive.
- Complete the addition of all terms.

Adding and subtracting three integers.

Example 3:

\[ (+2) - (+15) + (-22) = ? \]

- Step 1: Change the subtraction signs (–) to addition (+).
- Step 2: Switch the sign attached to the term that follows the operation.

\[ (+2) - (+15) + (-22) = (+2) + (-15) + (-22) \]

Remember: Change only the term that follows the operation you just changed.
Step 3: Complete the addition of all terms.

\((+2) + (-15) + (-22) = (-13) + (-22)\)

\((-13) + (-22) = (-35)\)

The Solution: \((+2) - (+15) + (-22) = (-35)\)

or, simplified:

\(2 - 15 - 22 = -35\)

Example 4:

\((+17) - (+24) + (-1) - (+6) = ?\)

Step 1: Change the subtraction signs \((-)\) to addition \(+)\).

Step 2: Switch the sign attached to the term that follows the operation.

\((+17) - (+24) + (-1) - (+6) = (+17) + (-24) + (-1) + (-6)\)

Step 3: Complete the addition of all terms.

\((+17) + (-24) + (-1) + (-6) = (-7) + (-1) + (-6)\)

\((-7) + (-1) + (-6) = (-8) + (-6)\)

\((-8) + (-6) = (-14)\)

The Solution: \((+17) - (+24) + (-1) - (+6) = (-14)\)

or, simplified

\(17 - 24 - 1 - 6 = -14\)
Simplifying Signed-Integer Expressions for Addition and Subtraction

You have been seeing a lot of parentheses in this lesson. Pre-algebra teachers and textbooks tend to "overuse" the parentheses in order to clarify the different ways plus and minus signs are used. An expression such as \(( + 8) – ( + 6) + (– 2)\) is really very cumbersome, but it clearly say + 6 is to be subtracted from + 8, and the result is added to – 2. Once you have mastered the concepts of adding and subtracting positive and negative numbers, you can simplify these expressions—and without changing their meaning.

Here are some simple examples of removing unnecessary parentheses:

\((+ 2)\) is the same as 2

\((– 2)\) is the same as – 2

\((+ 2) + (+ 3)\) is the same as 2 + 3

\((+ 2) – (+ 3)\) is the same as 2 – 3

\((– 2) – (+ 3)\) is the same as –2 – 3

\((+ 2) – (– 3)\) is the same as 2 + 3
Adding and Subtracting Signed Integers Exercises

1. \((+6) + (5) + (4) + (8) = \)

2. \((+8) - (+1) + (+4) + (-2) + (-5) = \)

3. \((+7) - (+5) + (-1) = \)

4. \((+7) + (+4) - (+6) = \)

5. \((+6) - (+1) + (-2) - (-9) = \)

6. \((-2) + (-1) - (-6) = \)

7. \((-6) - (+7) - (+1) + (+3) = \)

8. \((+3) - (+7) - (+8) - (+2) - (+3) = \)
7. Multiplying Signed Integers

The basic procedure for multiplying integers is identical to multiplying whole-number values. The only significant difference is dealing with the + and – signs that are assigned to the integer values.

The procedure for multiplying signed integers is:

• Step 1: Multiply the absolute value of the factors.
• Step 2: Give the appropriate sign to the product:
  Positive if the both factors have the same sign.
  Negative if the factors have opposite signs.
  Note: Zero has no sign.

Multiplying Integers Having the Same Sign

When the factors have the same sign—both positive or both negative—the product is always positive.

Multiply the two factors, disregarding the signs. Show the product as a positive integer.

Notice that when the signs of the two factors are the same, the product is positive.

Example 1:

\[ (+5) \times (+2) = \]

• Multiply the absolute value of the terms.

\[ |+5| \times |+2| = 10 \]

• Assign the appropriate sign to the product. Signs are the same, so the sign of the product is +: \[ +10 \]

Solution: \[ (+5) \times (+2) = (+10) \] or simply \[ 5 \times 2 = 10, \]
which looks exactly like multiplication for whole numbers.
Example 2:

\[( - 8) \times ( - 3) = \_\_\_\_\_\_\_\_\_\_\_\_\_\]

• Multiply the absolute value of the terms.

\[| - 8 | \times | - 3 | = 24\]

• Assign the appropriate sign to the product.

Signs are identical, so the sign of the product is +:      +24

Solution: \(( - 8) \times ( - 3) = ( +24)\) or simply \(- 8 \times - 3 = 24\).

Multiplying Integers Having Opposite Signs

When the factors have the opposite sign—one is positive an the other is negative—the product is always negative.

Multiply the factors, disregarding the signs.

Show the product as a negative integer.

Notice that when the signs of the two factors are different, the product is negative.

Example 3:

\[( - 7) \times ( + 2) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\]

• Multiply the absolute value of the terms.

\[| - 7 | \times | + 2 | = 14\]

• Assign the appropriate sign to the product.

Signs are opposite, so the sign of the product is –:      – 14

Solution: \(( - 7) \times ( + 2) = ( - 14)\) or simply \(- 7 \times 2 = -14\)
Lesson Summary

To multiply integers that have the same sign (both positive or both negative):

• Multiply the two factors, disregarding the signs.
• Show the product as a positive integer.

To multiply integers that have opposite signs:

• Multiply the factors, disregarding the signs.
• Show the product as a negative integer.

8. Dividing Signed Integers

The procedure for dividing signed integers is basically identical to the procedure for multiplying them:

• Step 1: Divide the absolute value of the terms.

• Step 2: Give the appropriate sign to the quotient.
  Positive if the terms both have the same sign.
  Negative if the terms have opposite signs.

Terminology for the division of signed integers.

Note: There is no difference between the way you should handle the signs for multiplication and division--
- positive result for same signs,
- negative result for opposite signs.

Dividing Integers Having the Same Sign

When the divisor and dividend have the same sign—both positive or both negative—the quotient is always positive.

So, divide the two terms, disregarding the signs and show the quotient as a positive integer.
Example 1:

\[(+ 14) \div (+ 2) = \underline{\text{____}}\]

- Divide the absolute value of the terms.
  
  \[|+14| \div |+2| = 7\]

- Assign the appropriate sign to the quotient.
  Both terms are positive, so the result is positive:  +7

Solution: \((+ 14) \div (+ 2) = +7\)  or more simply \(14 \div 2 = 7\)

Example 2:

\[(- 24) \div (- 8) = \underline{\text{____}}\]

- Divide the absolute value of the terms.
  
  \[|-24| \div |-8| = 3\]

- Assign the appropriate sign to the quotient.
  Both terms are negative, so the result is positive:  +3

Solution: \((- 24) \div (- 8) = (+3)\)  or, more simply \(- 24 \div (- 8) = 3\).

**Dividing Integers Having Opposite Signs**

When the divisor and dividend have the opposite sign—one is positive and the other is negative—**the quotient is always negative.**

So, divide the absolute values of the terms and show the quotient as a negative integer.
Example 3:

\[(+32) ÷ (–8) = \_\_\_\_\_\_\_\_
\]

- Divide the absolute value of the terms.

\[|+32| ÷ |-8| = 4\]

- Assign the appropriate sign to the quotient.
The terms have opposite signs, so the result is negative: -4

Solution: \((+32) ÷ (–8) = (–4)\)
or, more simply: \(32 ÷ (–8) = –4\)

**Integer Lesson Summary**

To divide integers that have the same sign (both positive or both negative):

- Divide the integers, disregarding the signs.
- Show the quotient as a positive integer.

To divide integers that have opposite signs:

- Divide the integers, disregarding the signs.
- Show the quotient as a negative integer.

9. **Combining Integer Multiplication and Division**

Most of the examples and exercises in these lessons deal with only two terms at a time. However, you don't have to advance much further in your study of mathematics until you encounter situation where you must multiply and divide strings of three or more signed integers. For instance:

\[(+4) x (+5) x (+12) x (+6) = ?\]

\[(+12) x (–10) ÷ (–8) = ?\]
Performing Combinations of Multiplication and Division

Important:

When multiplying and dividing groups of three or more integers, always perform the operations from left to right.

Example 1:

\[
(+ 12) \times (+ 2) \div (- 8) = ?
\]

- Multiply or divide the terms two at a time, from left to right.

\[
(+ 12) \times (+ 2) \div (- 8) = (+ 24) \div (-8)
\]

\[
(+ 24) \div (-8) = (-3)
\]

The Solution: \((+ 12) \times (+ 2) \div (- 8) = (-3)\) or \(-3\)

Example 2:

\[
(+ 16) \div (-8) \times (-6) = ?
\]

- Multiply or divide the terms two at a time, from left to right.

\[
(+ 16) \div (-8) \times (-6) = (-2) \times (-6)
\]

\[
(-2) \times (-6) = (+12)
\]

The Solution: \((+ 16) \div (-8) \times (-6) = (+12)\) or \(12\)
Multiplying and Dividing Signed Integers Exercises

Complete the following multiplication and division problems.

1. \( 2 \times 4 \times (-1) = \)

2. \(-4 \times 2 \times (-6) = \)

3. \(8 \cdot 6 \cdot 4 \cdot -2 = \)

4. \((4)(5)(-2)(2) = \)

5. \(2 \times 6 \div 3 = \)

6. \(2 \times 4 \div (-1) = \)

7. \(64 \div (16) \times (6) = \)

8. \(4 \div (2)(12) \div -8 = \)
10. Introducing Exponents

Recall that multiplication is a short-cut method for adding a group of equal numbers. For example:

\[3 + 3 + 3 + 3 = 4 \times 3 = 12\]

\[5 + 5 = 2 \times 5 = 10\]

The same idea applies to multiplying groups of equal numbers such as \(2 \times 2 \times 2 \times 2\) and \(12 \times 12\). The short-cut method in this case uses exponential notation:

\[2 \times 2 \times 2 \times 2 = 16\] expressed in exponential notation is \(2^4 = 16\)

\[12 \times 12 = 144\] expressed in exponential notation is \(12^2 = 144\)

We sometimes say a number is "raised to a certain power". For example, \(2^6\) could be spoken as "two raised to the sixth power." Or we could say "two to the sixth power."

**Powers of 2, or "Squares"**

By far, the most common exponent is 2. For instance, \(3^2\), \(5^2\), \(10^2\). Numbers that are raised to a power of two are said to be **squared**.

Examples:

"Three squared equals nine" \(3^2 = 9\) (same as \(3 \times 3\))

"Five squared equals twenty five" \(5^2 = 25\) (same as \(5 \times 5\))

"Ten squared equals one hundred" \(10^2 = 100\) (same as \(10 \times 10\))

\(13^2 = 169\)

**Powers of 3, or "Cubes"**

Another common exponent is 3. For instance, \(3^3\), \(5^3\), \(10^3\). Numbers that are raised to a power of three are said to be **cubed**.
Examples:

"Two cubed equals eight"  \[ 2^3 = 8 \]  (same as 2 \times 2 \times 2)
"Three cubed equals twenty seven"  \[ 3^3 = 27 \]  (same as 3 \times 3 \times 3)
"Ten cubed equals one thousand"  \[ 10^3 = 1000 \]  (same as 10 \times 10 \times 10)

**Note:** There are four special cases that you must remember when working with exponents.

**Special Cases**

0 raised to any power equals 0.
\[ 0^2 = 0 \]  or  \[ 0^9 = 0 \]

1 raised to any power equals 1.
\[ 1^3 = 1 \]

Any value raised to the 0 power is equal to 1.
\[ 2^0 = 1 \]  (This one is a little confusing, but that is the rule for the 0 exponent.)

Any value raised to the 1 power is equal to itself.
\[ 5^1 = 5 \]

**Exponents of Signed Integers**

The sign of any squared value is always positive.

Example:

The square of a positive number is a positive value.
\[ 3^2 = 3 \times 3 = 9 \]

The square of a negative number is a positive value.
\[ (– 4)^2 = (– 4)(– 4) = 16 \]
11. Ordering Operations With Integers

In Algebra, you often have equations that contain all of the primary operations. In such cases, follow these basic rules:

• When solving combinations of addition and subtraction operations on three or more terms, do the operations from left to right.

Example:

\[ 2 + 5 - 7 + 8 = 7 - 7 + 8 = 0 + 8 = 8 \]

• When solving combinations of multiplication and division operations on three or more terms, do the operations from left to right.

Example:

\[ 2 \times 12 \div 4 \times 8 = 24 \div 4 \times 8 = 6 \times 8 = 48 \]

Order of Precedence

But what about instances where you have combinations of addition, subtraction, multiplication, and division in the same expression? Solve from left to right? Not exactly. Then what about expressions that are enclosed in parentheses? What about terms with exponents? There are very specific rules for solving all these combinations of math operations--these rules are called order of operation, or order of precedence.
Solving Combinations of Addition, Subtraction, Multiplication, and Division

When solving combinations of addition, subtraction, multiplication, and division in the same expression:

- Do the multiplication and division first, from left to right.
- Do the addition and subtraction last, from left to right.

Examples:

1. Simplify $4 + 2 \times 6$
   The Problem: $4 + 2 \times 6 = ?$
   Multiply first: $4 + 2 \times 6 = 4 + 12$
   Add last: $4 + 12 = 16$
   The Solution: $4 + (2 \times 6) = 16$
   Notice that the solution is NOT $(4 + 2) \times 6 = 36$.

2. Simplify $6 + 18 \div 6$
   The Problem: $6 + 18 \div 6 = ?$
   Divide first: $6 + 18 \div 6 = 6 + 3$
   Add last: $6 + 3 = 9$
   The Solution: $6 + (18 \div 6) = 9$
   Notice that the solution is NOT $(6 + 18) \div 6 = 4$.

3. Simplify $4 + 3 \times 6 - 4 + 8 \times 2$
   The Problem: $4 + 3 \times 6 - 4 + 8 \times 2 = ?$
   Multiply first, from left to right:
   $4 + (3 \times 6) - 4 + (8 \times 2) = 4 + 18 - 4 + 16$
   Add/subtract last, from left to right:
   $(4 + 18) - 4 + 16 = 22 - 4 + 16$
   $(22 - 4) + 16 = 18 + 16$
   $18 + 16 = 34$
   The Solution: $4 + 3 \times 6 - 4 + 8 \times 2 = 34$
The order of operation for combination problems is:

1st = Parentheses

2nd = Exponents

3rd = Multiplication, Division (left-to-right)

4th = Addition, Subtraction (left-to-right)

---

**Exponents and Order of Operation Practice Exercises**

Cite the value of these squared integers.

1. $2^2 = ?$
2. $3^3 = ?$
3. $5^2 = ?$
4. $8^2 = ?$
5. $3^4 = ?$

Simplify these equations, using the correct order of precedence.

6. $8 + 16 ÷ 4 - 6 + 2 \times 3 =$
7. $24 - 12 ÷ 6 + 4 + 2 \times 3 =$
8. $2 \times 4 + 18 - 33 ÷ 3 =$
12. **Introducing Power Notation**

Power notation, the method for indicating the power of a number, has two parts:

- The base indicates the number to be multiplied.
- The exponent indicates the number of times the base is to be multiplied.

For Example:

\[
3^2 = 3 \times 3 = 9
\]

\[
2^4 = 2 \times 2 \times 2 \times 2 = 16
\]

\[
4^5 = 4 \times 4 \times 4 \times 4 \times 4 = 1024
\]

<table>
<thead>
<tr>
<th>Notation</th>
<th>Explanation</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n^1 = n)</td>
<td>Any number with an exponent of 1 is equal to that number, itself.</td>
<td>(5^1 = 5)</td>
</tr>
<tr>
<td>(n^0 = 1)</td>
<td>Any number with an exponent of 0 is equal to 1.</td>
<td>(3^0 = 1)</td>
</tr>
<tr>
<td>(1^k = 1)</td>
<td>1 to any power is equal to 1.</td>
<td>(1^4 = 1)</td>
</tr>
<tr>
<td>(0^k = 0)</td>
<td>0 to any power is equal to 0.</td>
<td>(0^5 = 0)</td>
</tr>
<tr>
<td>(n^k = \frac{1}{n^k})</td>
<td>Any number with a negative exponent is equal to 1 divided by that number with a positive exponent.</td>
<td>(2^{-3} = \frac{1}{2^3} = 0.125)</td>
</tr>
</tbody>
</table>
13. **Introducing Square Roots**

The opposite of squaring a number is taking the square root. For example:

- The square of 4 is 16
- The square root of 16 is 4

The main parts of square-root expression are the radical sign and the radicand. The radical sign tells us to take the root of the radicand.

**Notes:**

\[ \sqrt{0} = 0 \]

Don't bother trying to find the square root of a negative number such as: \( \sqrt{-9} \).

The solution exists, but not in the real number system. Pre-algebra courses deal only with the real number system, so you aren't responsible for finding square roots of negative numbers.

The table below lists the Squares for integers between 1 and 9. The Square Roots column shows how we can use square roots to convert the squares back to their roots.

<table>
<thead>
<tr>
<th>Squares</th>
<th>Square Roots</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(^2) = 1</td>
<td>( \sqrt{1} = 1 )</td>
</tr>
<tr>
<td>2(^2) = 4</td>
<td>( \sqrt{4} = 2 )</td>
</tr>
<tr>
<td>3(^2) = 9</td>
<td>( \sqrt{9} = 3 )</td>
</tr>
<tr>
<td>4(^2) = 16</td>
<td>( \sqrt{16} = 4 )</td>
</tr>
<tr>
<td>5(^2) = 25</td>
<td>( \sqrt{25} = 5 )</td>
</tr>
<tr>
<td>6(^2) = 36</td>
<td>( \sqrt{36} = 6 )</td>
</tr>
<tr>
<td>7(^2) = 49</td>
<td>( \sqrt{49} = 7 )</td>
</tr>
<tr>
<td>8(^2) = 64</td>
<td>( \sqrt{64} = 8 )</td>
</tr>
<tr>
<td>9(^2) = 81</td>
<td>( \sqrt{81} = 9 )</td>
</tr>
<tr>
<td>10(^2) = 100</td>
<td>( \sqrt{100} = 10 )</td>
</tr>
</tbody>
</table>
Memorizing the squares and square roots of the basic integers (1-9) is fairly easy. However, you will notice that between $8^2 = 64$ and $9^2 = 81$, there are 16 other numbers (65 - 80). What if you were required to find the square root of 73? There is no whole number integer that when squared equals 73.

There is a complicated procedure for finding the square root of other numbers, but you will not be asked to learn it in this class. Learn to use the square root key on your calculator.

14. Introducing Expressions and Equations

Algebraic expressions and equations: this is where your work really begins to look and feel like algebra. You are about to begin working with letters of the alphabet as well as numbers and signs of operation. From now on, letters such as x, y, and z are just as common as numbers 1, 2, and 3.

So what do letters in algebra mean? Until now—in basic arithmetic—you work with numbers, each number having a specific value or meaning. A "2" is a 2, for example. A "2" is always a 2. It is never a 6 and it is never a 10. It's like that for all numbers. We still use regular arithmetic numbers in algebra, but we also use terms expressed in letters. In algebra, the letter x, for example, can stand for a lot of different values. We can set x equal to 2 in one problem, but then set it equal to 6 in another. The letters in algebra can stand for an endless variety of values and combinations of values. Letters in algebra can even represent other letters.

Compare these two expressions:

Arithmetic expression: $2 + 1$

Algebraic expression: $x + 1$

The arithmetic expressions tells us to add 1 to the value of 2. The algebraic expression, however, covers a lot more territory by telling us to add a 1 to any value we choose. So if we let $x = 2$ in the algebraic
expression, it becomes \(2 + 1\). If we let \(x = 5\), it becomes the same as \(5 + 1\). You can see that the algebraic version is a lot more flexible than the arithmetic version.

**Definitions**

- A specific numerical value (such as 2, 4, -6, \(\frac{3}{4}\)) is called a **constant**. The value is constant ... unchanging.

- An algebraic term (such as \(x\), \(y\), \(a\), \(b\), and so on) is called a **variable**. The value can be varied.

- When constants or variables are connected by operations (such as +, -, \(\times\), or \(\div\)), you have an expression.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Spoken</th>
<th>Meaning</th>
<th>Variable</th>
<th>Constant</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x + 3)</td>
<td>&quot;x plus 3&quot;</td>
<td>Add var. (x) and con. 3</td>
<td>(x)</td>
<td>3</td>
<td>One var &amp; one constant</td>
</tr>
<tr>
<td>(y - 7)</td>
<td>&quot;y minus 7&quot;</td>
<td>Subtract con. 7 from var. (y)</td>
<td>(y)</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>3(a)</td>
<td>&quot;Three times a&quot;</td>
<td>Multiply con. 3 by var. (a)</td>
<td>(a)</td>
<td>3</td>
<td>3(a) can also be ((3)(a)) or (3a) The &quot;(x)&quot; multiplication sign should not be used. Too easily confused with var. (x)</td>
</tr>
</tbody>
</table>

AIDT - Pre-Algebra - April 22, 2008
Examples Of Algebraic Expressions.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Spoken</th>
<th>Meaning</th>
<th>Variable</th>
<th>Constant</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>z/5</td>
<td>&quot;z divided by 5&quot;</td>
<td>Divide var. z by con. 5</td>
<td>z</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>5/z</td>
<td>&quot;5 divided by z&quot;</td>
<td>Divide con. 5 by var. z</td>
<td>z</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>r - 3x</td>
<td>&quot;r minus 3 times x&quot;</td>
<td>Multiply x by 3, then subtract</td>
<td>r and x</td>
<td>3</td>
<td>Two variables and one constant</td>
</tr>
</tbody>
</table>

Definition:

An equation is a statement of equality between two expressions. It consists of two sets of algebraic expressions separated by an equal sign.

The purpose of an equation is to express equality between the two expressions. And what is the real difference between an algebraic expression and an algebraic equations?

Simple:

An equation includes an equal sign (=) and an expression does not. An expression can include signs of operation, but not an equal sign.

Equations:

\[ x + 4 = 8 \]

\[ y + 6 = 8 + 5 \]

\[ x/3 + 5 = 4y - 7 \]
15. Evaluating Algebraic Expressions

An algebraic expression can be evaluated by:

- Step 1: Assigning specific numerical values to all of the variables.
- Step 2: Completing all the operations

**Example 1**

Evaluate the expression $x + 9$ when $x = 5$

**Procedure:**

Replace the given value of $x$ in the expression.
$x + 9 = 5 + 9$

**Complete the operation:**
$5 + 9 = 14$

**Solution:**
$x + 9 = 14$
when $x = 5$.

**Example 2**

Evaluate the expression $11 - y$ when $y = 8$

**Procedure:**

Replace the given value of $y$ in the expression.
$11 - y = 11 - 8$

**Complete the operation:**
$11 - 8 = 3$

**Solution**
$11 - y = 3$
when $y = 8$
16. **Combining Like Terms**

Like terms are expressions that have the same variable, or letter. Some examples of like terms are:

2x and 4x, y and 5y,

**Examples:**

Combine like terms

1. 2x + 4x = 6x
2. 3x + 2x + 4 = 5x + 4
3. y + 3y + 2x = 4y + 2x

Notice that you can only combine the "like terms", that is, the ones that have the same variable. You can't combine 3 apples + 4 apples + 6 oranges and get 13 apples - you get 7 apples and 6 oranges.

This is an important rule to keep in mind when evaluating algebraic expressions or equations.

17. **Using The Distributive Property**

The distributive property of algebra deals with the combinations of multiplication and addition, or multiplication and subtraction. It allows you to remove parentheses from the expression and simplify the equation. The standard rule for the distributive property in addition is $a(b + c) = ab + ac$. For subtraction, the rule is $a(b-c) = ab - ac$.

This means that the multiplier "a" is distributed across both variables in the parentheses - "a times b, plus a times c", or "a times b, minus a times c". If you know the value of each of the variables, the problem is easily simplified by substituting the values and performing the operations.
For example:

Evaluate \(a(b + c)\), where \(a=3, b=5, c=6\)

**Step 1:** Substitute values

\[3(5 + 6)\]

**Step 2:** Do the operations

\[3(5 + 6) = 3 \times 5 + 3 \times 6\]

(Do the multiplication first)

\[3 \times 5 + 3 \times 6 = 15 + 18\]

\[15 + 18 = 33\]

18. **Solving Equations**

An equation is a mathematical statement of equality between two expressions.

Example: \(2x + 3 = 11\) is an equation

**Note:**

An equal sign is used for indicating equality between two expressions.

**Solving Equations of the Form** \(a + b = c\) **and** \(a − b = c\)

Here is an example of an equation of form \(a − b = c\)

\[x − 2 = 8\]

**Note:**

Don't be confused by the use of the variable "\(x\)" in this example. The form "\(a - b = c\)" does not mean you have to use \(a, b,\) and \(c\) - it is just saying "variable minus variable = variable".
The strategy for solving this equation is to do whatever is necessary to make variable $x$ stand alone on the left side of this equal sign.

This means getting rid of the $-2$ term. And how do we make a $-2$ go away? We add +2 to it: $-2 + 2 = 0$. That's zero ... the $-2$ is gone, and the $x$ variable stands alone on the left side of the equation. But remember the cardinal rule of algebra: Whatever is done to one side of the equal sign must also be done to the other side. In this example, we added 2 to the left side of the equal sign, so we must follow this by adding 2 to the right side.

$$x - 2 + 2 = 8 + 2$$

Then we clean up the equation by combining like terms:

$$x - 2 + 2 = 8 + 2$$
$$x + 0 = 10$$

The result is $x = 10$

Here is an example of an equation of form $a + b = c$

$$x + 2 = 8$$

The strategy for solving this equation is to do whatever is necessary to make variable $x$ stand alone on the left side of this equal sign. This means getting rid of the $+2$ term. And how do we make a $+2$ go away? We subtract 2 from it: $2 - 2 = 0$. That's zero ... the 2 is gone, and the $x$ variable stands alone on the left side of the equation. But remember the cardinal rule of algebra: Whatever is done to one side of the equal sign must also be done to the other side. In this example, we subtracted 2 from the left side, so we must follow this by subtracting 2 from the right side.

$$x + 2 - 2 = 8 - 2$$

Then we clean up the equation by combining like terms:

\[ x + 2 - 2 = 8 - 2 \]
\[ x + 0 = 6 \]

The result is \( x = 6 \)
Expression and Equation Exercises

1. **The number** $3^5$ **means:**
   a. 3 times 5  $(3 \times 5)$
   b. 3 divided by 5  $(3 \div 5)$
   c. 3 to the fifth power $(3 \times 3 \times 3 \times 3 \times 3)$
   d. 3 minus 5  $(3 - 5)$

2. **In Exercise 1 above,**
   a. The number 3 is called the ____________.
   b. The number 5 is called the ____________.

3. **True or False:**
   a. ___ The number 9 is the square of 3.
   b. ___ The number 9 is the square root of 18.
   c. ___ The number 9 is the square root of 81.
   d. ___ The number $9^0 = 1$

4. **The difference between an algebraic expression and an algebraic equation is:**
   a. the algebraic expression has no letters (variables) in it.
   b. the algebraic equation contains an equal (=) sign.
   c. the algebraic expression can be evaluated (worked out, solved).
   d. the algebraic equation contains no numbers.

5. **Combine the like terms in the following expressions:**
   a. $4a + a + 2a - 8 =$ ______________
   b. $6x - 2x + 3 - 1 =$ ______________
   c. $3x + 7x - 4y =$ ______________
   d. $12 + 2 + 5y - 2x =$ ______________

6. **Use the distributive property to evaluate:**
   $a(b + c)$, where $a=2$, $b=4$, $c=7$
   the correct answer is:
   a. 13
   b. 15
   c. 56
   d. 22
Remembering the cardinal rule of algebra that "Whatever is done to one side of the equal sign must also be done to the other side", solve the following equations. Write out the steps

7. \( y - 5 = 13 \)

8. \( 2x + 11 = 43 \)

9. \( 4a - 9 = 15 \)

10. \( x(3 + 4) - 8 = 20 \)
Lesson Summary:

Algebra can be very difficult for those who do not have a good understanding of the preceding basic terminology and rules. As you learn more about algebra, you start to see how it has unlimited application in everything we do.

This lesson was designed to try to break the impression that most people have about algebra. It does not have to be difficult, if you just take the time to learn the rules and remember that the equal sign means "everything on the left side has the same value as everything on the right side". So substitute, distribute and simplify all you want - just keep it balanced and equal.